

NAG Toolbox for MATLAB

s10aa

1 Purpose

s10aa returns a value for the hyperbolic tangent, $\tanh x$, via the function name.

2 Syntax

```
[result, ifail] = s10aa(x)
```

3 Description

s10aa calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x \sum_{r=0}' a_r T_r(t)$$

where $-1 \leq x \leq 1$, $-1 \leq t \leq 1$, and $t = 2x^2 - 1$.

For $1 < |x| < E_1$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For $|x| \geq E_1$, $\tanh x = \text{sign } x$ to within the representation accuracy of the machine and so this approximation is used.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{2x}{\sinh 2x} \delta \right|.$$

That is, a relative error in the argument, x , is amplified by a factor approximately $\frac{2x}{\sinh 2x}$, in the result.

The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

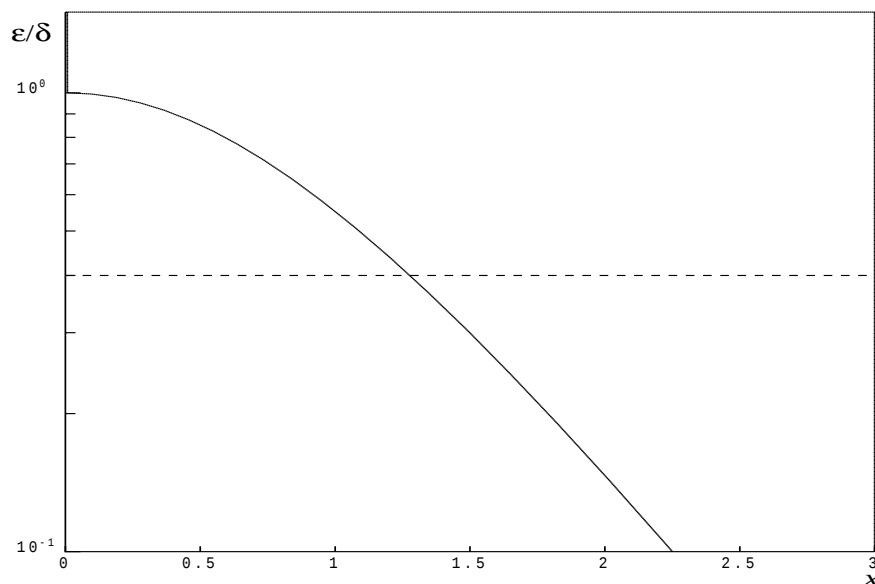


Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the precision of machine representation.

8 Further Comments

None.

9 Example

```
x = -20;
[result, ifail] = s10aa(x)

result =
    -1
ifail =
     0
```

